

TeV scale unification in four dimensions versus extra dimensions

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Abstract

The gauge coupling constant unification at the low ($O(\text{TeV})$) scale can be obtained just in four dimensions, without help of the power like renormalization group evolution in extra dimensions, due to the presence of some extra particle states at intermediate scales. We show explicit examples of such extra states in the range of 100 GeV – 1 TeV which can be easily observed on future colliders and can have important impact on the particle phenomenology. The problems of the low scale grand unification and proton stability are also discussed.

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1 Introduction

The gauge hierarchy problem is one of the most challenging problem in particle physics. It questions why the electroweak scale $M_W \sim 100$ GeV is so small as compared to the scale of gravitational interaction $M_P \sim 10^{18}$ GeV which is often considered as a fundamental scale of physics. Recently, however, it was shown in ref. [1, 2] that the essence of the hierarchy problem can be drastically changed in the presence of reasonably large extra dimensions. In particular, in the context of D -dimensional theory, with 1 time and $D - 1$ spatial coordinates, one can consider a picture when the Standard Model (SM) particles are localized on a 3-brane identified with the observed 3-dimensional space, while gravity propagates in the full D -dimensional bulk with $N = D - 4$ compact dimensions. In this situation the fundamental scale of gravitational interaction could be as low as $M_{Pf} \sim \text{few TeV}$, whereas the observed weakness of the Newtonian constant is due to the large size of extra dimensions ($R \gg \text{TeV}^{-1}$). The effective Planck scale M_P of the 4-dimensional theory is related to M_{Pf} via the large volume $V \sim R^N$ of the internal space as $M_P^2 \simeq M_{Pf}^{2+N} R^N$. Hence, the hierarchy problem between M_W and M_{Pf} is "nullified" but now it can be reformulated as a question to why the extra dimensions have a size R much larger than fundamental Planck length M_{Pf}^{-1} . At distances larger than the typical size of these extra dimensions gravitational potential goes to its standard Newton's form. Moreover, all currently known experimental data including that of astrophysical as well as cosmological constraints can be barely satisfied by the above theoretical construction in the case of two or more extra dimensions, $N \geq 2$ [3].

Recent developments in string theory have revealed an interesting possibility that the string scale M_{str} may be much lower than the fundamental Planck scale M_P [6] and perhaps as low as $\sim \text{few TeV}$ [2, 8]. As it is well known, any consistent superstring theory has two parameters: a mass scale M_{str} and dimensionless string coupling g_{str} given by the vacuum expectation value (VEV) of the dilaton field. Upon the compactification of extra dimensions these parameters determine the four-dimensional Planck mass M_P and a single dimensionless gauge coupling g at the string scale M_{str} . Thus, in the context of string theory we have to achieve an unification of the SM gauge couplings at M_{str} , while they are known substantially differ from each other at currently available energies. However, the extrapolation of gauge couplings from their precisely measured values at Z -peak to higher energies according to the ordinary 4-dimensional renormalization flow gives the unification scale around 10^{16} GeV, much higher than the electroweak scale.⁴

One possibility to achieve a low-scale ($O(\text{TeV})$) gauge coupling unification is to consider the possibility that the number of space-time dimensions experienced by the SM fields are risen when the SM gauge couplings are running on their way to the unification point, i.e. to assume the presence of some extra dimensions with radii larger than M_{str} . This leads to change in the evolution of the gauge couplings from 4-dimensional logarithmic to higher-dimensional power-low and, hence, accelerates unification in an energy region where the theory becomes high dimensional. Considerable interest to this possibility was renewed by the first indication of unification of gauge couplings extrapolated from one-loop calculation [8]. After more accurate test of the minimal scenario it became evident that in order to justify the unification condition [9] an extension of the SM or the Minimal Supersymmetric Standard Model (MSSM) is required. Typically, to improve the unification picture of the minimal scenario, one considers the models with

⁴ This is an additional point of the problem of hierarchy of fundamental scales.

extra vector-like matter [10, 11] or extended SM gauge symmetry [12] or when the SUSY breaking scale is large than the compactification scale [13], but both are order of TeV.

In this connection the following question might be naturally raised: *What kind of extension of the SM or MSSM are needed to achieve the low-scale gauge coupling unification just in four dimensions without help of extra dimensions?* If such models can be constructed then they can be also considered from the point of view of a possible resolution of the unification problem in various higher dimensional models where the SM fields are restricted to be stuck on a 3-brane and thus do not feel the extra dimensions at all. This might be the situation within the millimeter size extra dimensions [1, 2] or within the models with non-compact extra dimensions [14].

In this paper we study systematically the group-theoretical constraints on the gauge coupling b -functions that ensure TeV scale unification in the SUSY as well as in the non-SUSY cases. We find a certain set of new particles which modify properly the running of the gauge couplings driving them to unify at energies around the TeV scale. It is remarkable that such particles can be easily observed at future colliders.

The paper is organized as follows. In the next section we present a general analysis of the gauge coupling unification and determine how the corresponding b -factors should be modified in order to have successful low-scale unification. Based on this analysis we give some explicit examples of extra matter multiplets which provide the gauge coupling unification at the two-loop level. In Section 3 we briefly discuss some possibilities how to circumvent the proton decay problem which we face in low-scale GUT models. Finally, in Section 4, we present our conclusions.

2 Gauge coupling unification at TeV scale

Let us start by considering general aspects of the gauge coupling unification at one-loop level. The coupling constants $g_{3,2,1}$ of the standard gauge factors $SU(3) \times SU(2) \times U(1)$ at low energies are known with a quite good accuracy. Namely, the world averages for the values $\alpha_i = g_i^2/4\pi$ at Z -peak are the following [15]:

$$\alpha_1^{-1}(M_Z) = 58.98 \pm 0.04, \quad \alpha_2^{-1}(M_Z) = 29.57 \pm 0.03, \quad \alpha_3(M_Z) = 0.119 \pm 0.002. \quad (1)$$

The running constants $\alpha_i(\mu)$ at higher energies, $\mu > M_Z$, can be calculated by the standard renormalization group (RG) equations. The fact of the gauge coupling unification means that the all three running constants become equal at some scale $\mu = M_U$, i.e. $\alpha_1(M_U) = \alpha_2(M_U) = \alpha_3(M_U) = \alpha_U$. In principle, starting from some scale $M > M_Z$, the theory may include some extra particle states F in non-trivial representations of $SU(3) \times SU(2) \times U(1)$, with masses $M_F \geq M$. In this case, the one-loop RG equations relating α_U to $\alpha_i(M)$ and $\alpha_i(M_Z)$ read as:

$$\alpha_i^{-1}(M) = \alpha_U^{-1} + \frac{b_i^S}{2\pi} \ln \frac{M_U}{M} + \sum_F \frac{b_i^F}{2\pi} \ln \frac{M_U}{M_F}, \quad (2)$$

and

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M) + \frac{b_i^S}{2\pi} \ln \frac{M}{M_Z} \quad (3)$$

where b_i^S are the standard one-loop b -coefficients. Namely, in the Standard Model we have $b_i^S = b_i^{\text{SM}} = (41/10, -19/6, -7)$, while in the MSSM $b_i^S = b_i^{\text{MSSM}} = (33/5, 1, -3)$.⁵ The second term in (2) stands for the contribution of extra particles F , with one-loop coefficients b_i^F which depend on the representation content of the latter.

One can introduce the effective coefficients b_i which extrapolate eqs. (2) as

$$\alpha_i^{-1}(M) = \alpha_U^{-1} + \frac{b_i}{2\pi} \ln \frac{M_U}{M}. \quad (4)$$

From here follows that

$$\frac{\alpha_i^{-1}(M) - \alpha_j^{-1}(M)}{\alpha_j^{-1}(M) - \alpha_k^{-1}(M)} = \frac{b_i - b_j}{b_j - b_k}, \quad i, j, k = 1, 2, 3, \quad (5)$$

and for the difference of the effective b -coefficients $B_{ij} = b_i - b_j$ we obtain:

$$B_{ij} = \frac{B_{ij}^S + \sum_F B_{ij}^F}{1 + \frac{1}{2\pi A_{ij}(M)} \sum_F B_{ij}^F \ln \frac{M_F}{M}}, \quad (6)$$

where $B_{ij}^{S,F} = b_i^{S,F} - b_j^{S,F}$ and $A_{ij}(M) = \alpha_i^{-1}(M) - \alpha_j^{-1}(M)$. As for the unification scale and the unified gauge constant, we have respectively:

$$\ln \frac{M_U}{M} = \frac{2\pi A_{ij}^{-1}(M)}{B_{ij}}, \quad \alpha_U^{-1} = \alpha_1^{-1}(M) - A_{12}(M) \frac{b_1}{B_{12}} \quad (7)$$

Hence, the criterion for the gauge coupling crossing at one scale is encoded into the condition:

$$\frac{B_{12}}{B_{23}} = \frac{A_{12}(M)}{A_{23}(M)} \equiv R(M), \quad (8)$$

In other words, the theoretical ratio $B \equiv B_{12}/B_{23}$ which depends on the extra particle content and their mass splitting, should coincide with the quantity $R(M)$ which is determined by the experimental values of the gauge coupling constants (1). In particular, for $M = M_Z$, we have $R(M_Z) = 1.39 \pm 0.03$, with the uncertainty corresponding to 2σ error-bars in eqs. (1). For M larger than M_Z , we obtain from (3):

$$R(M) = R(M_Z) \frac{1 - \frac{B_{12}^S}{2\pi A_{12}(M_Z)} \ln \frac{M}{M_Z}}{1 - \frac{B_{23}^S}{2\pi A_{23}(M_Z)} \ln \frac{M}{M_Z}} \quad (9)$$

Therefore, for M in the range up to TeV or so the values of $R(M)$ remain very close to $R(M_Z)$ (see Fig. 1). In particular, in the MSSM we have $R(M) = R(M_Z)$ with a very good accuracy while in the SM we obtain $R(M) = R(M_Z)[1 - 10^{-2} \ln(M/M_Z)]$, where the correction is comparable to uncertainty in $R(M_Z)$ itself. The values of $R(M)$ for $M > M_Z$ calculated in 1 and 2-loops are shown in Fig. 2.

In particular, in the SM case, without the extra particle contribution, the unification condition is not satisfied: we have $B = B_{12}^S/B_{23}^S \approx 1.90$, about 17 standard deviations away from the range $R(M_Z) = 1.39 \pm 0.03$. However, in the MSSM we have $B = 1.4$, in a wonderful agreement with $R(M_Z)$. In this way, we have once again demonstrated the

⁵ In the following, for the sake of definiteness, we take the effective SUSY scale as M_Z .

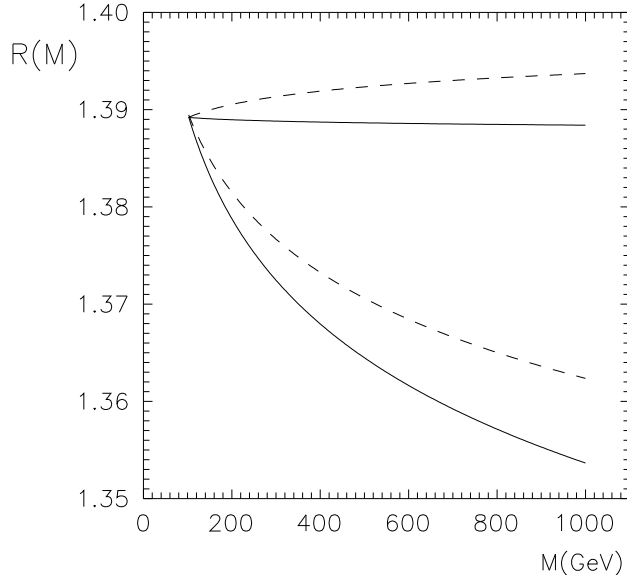


Figure 1: R as a function of the intermediate scale M for the SM (lower solid) and MSSM (upper solid) at one loop. Dash curves correspond to the 2-loop results.

remarkable success of the supersymmetric grand unification [16]. Since the MSSM yields rather small values of B_{ij} , namely $B_{12}^S = 5.6$ and $B_{23}^S = 4$, the gauge coupling unification occurs at very large scale, $M_U \simeq 10^{16}$ GeV, which also renders the proton to be enough long-living.

On the other hand, lower M_U would need the larger values of the coefficients B_{ij} which should be achieved due to the contribution of the extra particles. In particular, for achieving the unification at the scale M_U order few TeV one has to choose the representation content of the extra states F so that the coefficients B_{12} and B_{23} are large, $O(100)$, and they are related as in eq. (8). The required correlation $B_{23} = B_{12}/R(M_Z)$ as well as the values of the unification scale as a function of B_{12} is shown in Fig. 2.

In order to define better the rules of the game, let us assume that extra particles may have masses from few hundred GeV to few TeV. In the standard model context they can present in the form of new scalars or vector-like fermions. In the MSSM context one has to introduce chiral superfields in vector-like or self-adjoint representations. Such massive states would not affect the phenomenology of the standard model and also would satisfy the anomaly cancellation conditions.

Clearly, for achieving the big values of B_{23} in more economic way, one has to increase b_2 without increasing much b_3 . In other words, one has to introduce as few as possible extra states with the non-trivial colour content, and ultimately one could restrict himself only by the colour singlet states. Moreover, too many colour states would violate the asymptotic freedom of $SU(3)$ so that the gauge coupling crossing could occur in the strong coupling regime where the perturbation theory is no more valid, or even in the unphysical region with the negative α_U^{-1} . In particular, in order the evolution of α_3^{-1} to higher energies does not change the slope to a negative value, we need $\sum_F b_3^F$ to be less than 7 in the case of the SM, or less than 3 in the MSSM context.

This simplifies our selection rules for the representation content of the extra states.

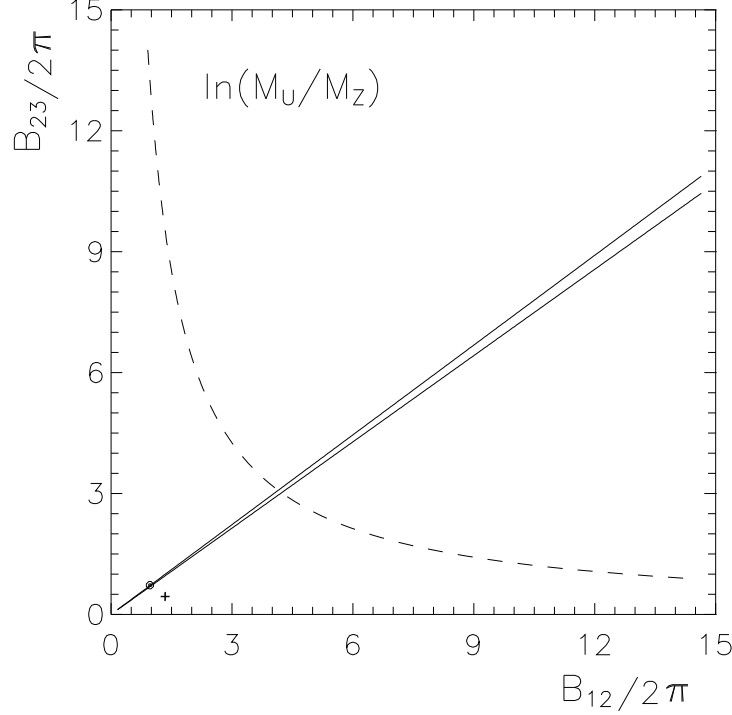


Figure 2: Thin area between solid lines indicates the correlation between the effective coefficients B_{12} and B_{23} required by the gauge coupling constant unification (8) for the case $M = M_Z$. The cross and bullet mark the values B_{12} , B_{23} respectively for the SM and MSSM cases. The dashed curve shows the unification scale as a function of B_{12} .

The low scale unification of the gauge couplings should be achieved essentially due to the contributions of the colour singlet extra states - sort of heavy leptons. We have also to demand that electric charges $Q = T_3 + Y/2$ of these states are integer, which means that the isospin T and hypercharge Y should be related as $Y = 2(T + k)$, k being any integer.

For definiteness, let us consider the supersymmetric case, with extra matter in the form of vector-like pairs of the colour singlet chiral superfields in $SU(2) \times U(1)$ representations $F(D, Y) + \bar{F}(D, -Y)$, where $D = 2T + 1$ is the dimension of $SU(2)$ representation. These fields should form massive states with the mass terms $M_F \bar{F} F$, $M_F \geq M_Z$.⁶

Each of the superfields F and \bar{F} contribute the one-loop b coefficients as follows:⁷

$$b_1^F = \frac{3}{20}DY^2, \quad b_2^F = \frac{1}{12}D(D^2 - 1), \quad b_3^F = 0 \quad (10)$$

Obviously, the existence of extra heavy states, with masses in the range from few hundred GeV to TeV can be directly tested in the future colliders. In addition, these states

⁶The superfields in real representations of $SU(2) \times U(1)$, i.e. the ones with $Y = 0$ and D being the odd number, can have also Majorana like mass terms $M_F \bar{F} F$.

⁷In principle, small amount of the colour triplets (the heavy exotic quarks) can be also allowed, in representations $Q(3, D, Y) + \bar{Q}(\bar{3}, D, -Y)$, with the hypercharges $Y = \frac{1}{3} + D + 2k$, k being any integer. Their contributions in the b coefficients are: $b_1^Q = \frac{9}{20}DY^2$, $b_2^Q = \frac{1}{4}D(D^2 - 1)$ and $b_3^Q = \frac{1}{2}D$. In order to maintain the asymptotic freedom of $SU(3)$, the $SU(2)$ dimension of these multiplets should not be very large, $\sum_Q D_Q < 3$.

may contain the particles with the same quantum numbers as ordinary leptons and quarks, which can also have impact for the mass generation mechanism of the latter, in the spirit of the mechanism [18]. Since the mass scale of the heavy states is close to the electroweak scale, then in general one could expect some violation of the universality (unitarity) of the CKM mixing and additional contributions to the flavour changing phenomena.

One can further simplify a situation and consider all extra particles having the same mass, $M_F = M$. In this case the eq. (6) reduces to $B_{ij} = B_{ij}^S + \sum_F B_{ij}^F$. Therefore, our goal is to select the representation content of the extra F particles so that the non-standard contributions in B_{12} and B_{23} are $O(100)$, and their ratio is $\approx R(M)$. In particular, one can choose the representations which predict $B = 1.4$, as it holds in the case of the MSSM.

Clearly, many different solutions can be envisaged. For example, one can consider extra states consisting exclusively of heavy replicas of the ordinary lepton species, $L(2, -1) + \bar{L}(2, 1)$ and $\bar{E}(1, 2) + E(1, -2)$, for simplicity all located at the same scale M . Then, if we take their multiplicities as $N_L = 2N$ and $N_E = 3N$, with N being any integer number, we obtain $b_1^{\text{NS}} = 4.8N$ and $b_2^{\text{NS}} = 2N$, and so $B^{\text{NS}} = 1.4$. On the other hand, for having the low scale unification, N should be large number. For example, for $M = 400$ GeV, the unification scale $M_U \simeq 10$ TeV can be obtained only if $n = 18$.⁸

Another possibility is to seek for just one big representation which could do a job alone. A candidate we have found is a vector-like multiplet $F(7, 8) + \bar{F}(7, -8)$. It leads to $b_1^F = 134.4$ and $b_2^F = 56$, and hence $B^{\text{NS}} = 1.4$. These coefficients are enough big, $B_{12} = 78.4$, to provide M_U order TeV. And finally, one can consider a mixed situation, containing some big representations appended by few small ones. Some possible candidates are given below.

A subtle questions which arise here is whether the solutions found in 1-loop approximation will be stable against 2-loop corrections. This question seems most challenging for the solutions with large representations in which case the 2-loop RG effects are expected to be significant. Therefore, one needs to examine the 2-loop RG equations:

$$\frac{d\alpha_i}{dt} = -\frac{b_i}{2\pi}\alpha_i^2 - \frac{1}{8\pi^2}\alpha_i^2 \sum_{j=1}^3 b_{ij}\alpha_j \quad (11)$$

where $b_i = b_i^S + \sum_F b_i^F$ are the 1-loop b -coefficients, with b_i^F given as in eq. (10), and $b_{ij} = b_{ij}^S + \sum_F b_{ij}^F$ are the 2-loop ones. In the MSSM the standard 2-loop coefficients are given by the matrix:

$$b_{ij}^S = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}, \quad (12)$$

while the non-standard contributions can be calculated as [17]:

$$\begin{aligned} b_{11}^F &= \frac{9}{100}DY^4, & b_{12}^F &= 3b_{21}^F = \frac{3}{20}D(D^2 - 1)Y^2, \\ b_{22}^F &= \frac{1}{12}(D^2 + 3)D(D^2 - 1), \end{aligned} \quad (13)$$

The two loop corrections significantly affect the solutions with small representations. However, a remarkable thing occurs for the solutions with large representations: the fact

⁸ Alternatively, one could take the same number of these species, $N_L = N_E$ but consider the mass splitting between these states. For instance, $M_U \simeq 10$ TeV can be achieved by taking 42 copies for each of $L + \bar{L}$ and $\bar{E} + E$, with $M_E = 200$ GeV and $M_L = 1$ TeV.

of the gauge coupling unification remains robust, whereas naively one would expect strong violations since the 2-loop effects normally become bigger with bigger representations.

The reason of the stability of the unification condition has a simple explanation. To demonstrate this, it is convenient to approximate the two-loop RG predictions as follows:

$$\alpha_i^{-1}(M) = \alpha_U^{-1} + \frac{b_i}{2\pi} \ln \frac{M_U}{M} + \frac{1}{4\pi} \sum_{j=1}^3 \frac{b_{ij}}{b_j} \ln \frac{\alpha_U}{\alpha_j(M)} \quad (14)$$

One can rewrite these equations in a form analogous to 1-loop extrapolation (4):

$$\alpha_i^{-1}(M) = \alpha_U^{-1} + \frac{b_i^{(2)}}{2\pi} \ln \frac{M_U}{M} \quad (15)$$

where

$$b_i^{(2)} = b_i + \frac{1}{2} \sum_{j=1}^3 \frac{b_{ij}}{b_j} X_j, \quad X_j = \frac{\ln(\alpha_U/\alpha_j(M))}{\ln(M_U/M)}. \quad (16)$$

Then the gauge coupling crossing condition becomes

$$B^{(2)} \equiv \frac{B_{12}^{(2)}}{B_{23}^{(2)}} = R(M). \quad (17)$$

Hence, the stability of the 1-loop solution with satisfying (8) implies that the ratio of 2-loop factors $B^{(2)} \equiv B_{12}^{(2)}/B_{23}^{(2)}$ should remain close to the ratio of 1-loop factors $B = B_{12}/B_{23}$.

This is what actually happens when the RG evolution of couplings is dominated by one big representation $F(D, Y)$, due to a conspiracy between the 1- and 2-loop coefficients (10) and (13). Indeed, one obtains:

$$B_{12}^{(2)F} = B_{12}^F \left[1 + \frac{2}{D}(X_1 + 3X_2) \right] - 2X_2, \quad B_{23}^{(2)F} = B_{23}^F \left[1 + \frac{2}{D}(X_1 + 3X_2) \right] + 2X_2 \quad (18)$$

and so $B^{(2)} \simeq B$, with about a per cent correction caused by the terms $2X_2$.

We found the solutions by solving numerically the precise 2-loop equations as well. Namely, first we justify the unification condition (8), by selecting the particle states with appropriate quantum numbers which could ensure the desired modification of b -factors at one-loop. Then we test these solutions by numerical analysis at two-loop level.⁹

As it was expected from the above analysis, the case of large multiplet $F(7, 8) + \overline{F}(7, -8)$ which satisfies the 1-loop criterion of the gauge coupling crossing (8), $B = 1.4$, is perfectly stable in two-loops. However, the value of unification scale changes significantly. Namely, by taking the mass scale $M = 400$ GeV, we had $M_U^{(1)} = 3.3$ TeV at 1-loop, while at 2-loops we obtain $M_U^{(2)} = 2$ TeV.

The list of some mixed solutions is given below. They all imply $B = 1.4$ at 1-loop and are stable against 2-loop RG analysis. We show the $SU(2) \times U(1)$ content and multiplicities of the chiral superfields (plus their conjugates) and the unification scales at 1- and 2-loops respectively.

$$\begin{aligned} (6, 7) + 3 \times (2, 1) + (1, 2) &: M_U^{(1)} = 8.2 \text{ TeV}, \quad M_U^{(2)} = 4.8 \text{ TeV} \\ 2 \times (5, 6) + 2 \times (3, 2) &: M_U^{(1)} = 4.6 \text{ TeV}, \quad M_U^{(2)} = 3.3 \text{ TeV} \\ 2 \times (4, 5) + 2 \times (3, 2) &: M_U^{(1)} = 21 \text{ TeV}, \quad M_U^{(2)} = 15 \text{ TeV} \end{aligned} \quad (19)$$

⁹ Higher-loop corrections are expected to be negligible as we have a logarithmic running of gauge couplings and all gauge couplings are in perturbative region.

In the case of solutions with only small representations $L + \bar{L}$ and $E + \bar{E}$, with $N_L = 36$ and $N_E = 54$, the coupling crossing does not occur anymore within 2σ error-bars in $\alpha_3(M_Z)$, and it requires about 4.5σ deviation. In order to obtain the coupling crossing at 2σ level, one has to change the number of fields, namely to take $N_L = 35$ and $N_E = 54$, which at one-loop would correspond to $B = 1.45$. On the other hand, the unification scale does not change substantially and remains around 10 TeV.

3 GUT picture and proton stability

An interesting issue is whether the coupling constant crossing can really correspond to the possibility that at the scale M_U of a few TeV three gauge factors $SU(3) \times SU(2) \times U(1)$ are indeed embedded in some grand unified group. The main obstacle for the realization of the consistent GUT scenario is the problem of proton stability. Indeed, if the quark and lepton fields are unified into the $SU(5)$ multiplets $\bar{5}_k$ and 10_k , $k = 1, 2, 3$, then the processes mediated by the heavy gauge bosons X and Y of $SU(5)$ with masses $\sim M_U$ will lead to the catastrophically fast proton decay.

Nevertheless, several ways out can be envisaged. One possible scenario for the suppression of the proton decay can be thought as follows. Suppose the gauge $SU(5)$ theory which is in the confining phase outside the 3+1-dimensional domain wall (3-brane) produced by some master field ϕ , a singlet of $SU(5)$. However on the 3-brane, due to the appropriate coupling with the master field ϕ , some adjoint scalar field Σ is triggered to develop the $SU(3) \times SU(2) \times U(1)$ preserving VEV, $\langle \Sigma \rangle = V \cdot \text{diag}(2, 2, 2, -3, -3)$. Then, according to the Dvali-Shifman mechanism [19], all gauge fields of the standard model are localized on the 3-brane, while the massive (on the 3-brane) X and Y bosons freely propagate in the bulk. Now, assuming also that chiral matter fields in $\bar{5} + 10$ representations of $SU(5)$ are confined on the 3-brane their effective 4-dimensional couplings to X and Y bosons could be suppressed by the volume-factor coming from the integration out of extra coordinates in the effective Lagrangian:

$$g_{X,Y}^{(4-dim)} = \frac{1}{\sqrt{V_N}} g_{SU(5)}^{(bulk)}, \quad (20)$$

where $V_N \sim R^N$ is the volume of compact internal space with N extra dimensions of radius R . The rough estimation shows that the case of $N = 2$ extra dimensions with $R \sim 10^{-3}$ mm is perfectly safe to satisfy experimental bounds on the proton lifetime even if the unification scale is around TeV.

The obvious trouble with the above mechanism is related with the colour triplet partner of the electroweak doublet. The point is that, as long as gluons are confined on the brane the coloured Higgs has to be restricted on the brane as well, due to the colour flux conservation argument [19]. This coloured Higgs (as well as its superpartner, Higgsino) can mediate fast proton decay, unless it is completely decoupled from the quarks and leptons. Such a decoupling can be indeed achieved in some extended GUTs (say, in $SO(10)$ GUT) thanks to the special pattern of the GUT symmetry breaking (for more details see [20]). Another way is to simply remove the fundamental Higgs from the theory, attributing the electroweak symmetry breaking to some dynamical mechanism involving say the top-antitop condensation [21].

Another possibility of keeping baryon and /or lepton numbers approximately conserved in 4-dimensions and thus suppressing the proton decay up to a desired level is offered by

the mechanism of ref. [22]. It reveals to the field-theoretic localization of chiral matter on a fat 3+1-dimensional domain wall (3-brane) when the quarks and leptons are localized at a different points along the extra coordinate. This mechanism is also compatible with GUT models, providing that GUT symmetry, say $SU(5)$, is broken down to the $SU(3) \times SU(2) \times U(1)$ by the VEV of the bulk adjoint field Σ . The higher-dimensional model is initially vector-like, so along with the usual $(\bar{5} + 10)$ representations for each family of ordinary quarks and leptons there are mirror fermions $(5 + \overline{10})$ as well. In the bulk the Dirac masses of the quarks and leptons residing in quintuplets and decuplets are splitted as a result of GUT symmetry breaking according to the following equations:

$$\bar{5}(\langle \Sigma \rangle + M_5) 5, \quad \overline{10}(\langle \Sigma \rangle + M_{10}) 10. \quad (21)$$

Here we omit Yukawa constants and family indices for simplicity; M_5 and M_{10} are $SU(5)$ invariant masses for the quintuplets and decuplets, respectively.

The matter fields above are assumed to couple with the master scalar field ($SU(5)$ -singlet) ϕ as well. This scalar field "produce" the domain wall $\phi(x_5) = \phi_0 \tanh(\mu x_5)$ with a certain thickness μ^{-1} . Then, as it is well known, only the chiral matter (e.g. $(\bar{5} + 10)$) get localized on the domain wall. The points where the wave functions of the quarks and leptons are peaked are actually different (schematically we denote them as x_5^q and x_5^l , respectively) due to the presence of the $SU(5)$ -breaking VEV of Σ in (21) and are determined by the equations:

$$\bar{5}(\langle \Sigma \rangle + M_5 + \phi(x_5)) 5 = 0, \quad \overline{10}(\langle \Sigma \rangle + M_{10} + \phi(x_5)) 10 = 0. \quad (22)$$

Thus, any transition between quarks and leptons in 4 dimensions will contain an exponential suppression factor $\exp(-\mu^2(x_5^q - x_5^l)^2)$, since the interactions are non-local in extra dimension. This exponential factor suitably adjusted could indeed suppress the proton decay up to a desired level.

Perhaps the most natural realization of such GUT models can be found within the higher-dimensional theories when certain symmetries inherited from the compactified internal space can be used to prevent the rapid proton decay. One of such intrinsically higher-dimensional mechanism has been proposed in [8] where the minimal SUSY $SU(5)$ was considered in 5 dimensions compactified on S^1/Z_2 orbifold. The chiral matter (i.e. ordinary quarks and leptons and their superpartners) in [8] is assumed to be located at the orbifold fixed point, while gauge fields and possibly Higgs fields as well are allowed to propagate in the full 5-dimensional bulk. Then, assuming that X and Y bosons (and the colored Higgs as well) are odd under the Z_2 orbifold parity, one can totally decouple them from the quarks and leptons, so that they could not be responsible for the proton decay¹⁰.

Finally, one could construct the GUT models with absolutely stable proton just in 4 dimensions. For some models constructed in the past see e.g. [24]. The obvious candidates for such a GUTs are the models with discretely covered GUT symmetry, such as $(SU(3))^3$ trinification or $SU(N) \otimes SU(N)$ -type theories with appropriately chosen matter representations. In such models typically there are no X and Y gauge bosons (or they are not responsible for the transition of ordinary quarks into leptons and vice

¹⁰ The minimal proposal of ref. [8] do not seem fully satisfactory since the scalar leptoquarks remained couple with quarks and leptons could also mediate unacceptable rapid proton decay. For further developments of the mechanism of ref. [8] see [23].

versa) and thus there is no gauge mediation of the proton decay. As to the coloured Higgs (Higgsino) mediation of proton decay one can still use one of the mechanism described above.

Moreover, one can construct the GUT models based on the simple groups as well. As an instructive example we scotch here $SU(5)$ GUT with special fermion assignment. Namely, imagine that the theory contains 5 states $\bar{F}_{1,2} \sim \bar{5}$ and $T_{1,2,3} \sim 10$, and 3 anti-states F and $\bar{T}_{1,2}$ per each generation, among the other possible vector-like states. Imagine also that $SU(5)$ symmetry is broken by the fields Φ and Ω , both in the reducible $24+1$ representations, having the following "orthogonal" VEV patterns: $\langle \Phi \rangle = V_1 \cdot \text{diag}(1, 1, 1, 0, 0)$ and $\langle \Omega \rangle = V_2 \cdot \text{diag}(0, 0, 0, 1, 1)$. There is also one extra $24+1$ field Σ which develops VEV of the form $\langle \Sigma \rangle = \sigma \cdot \text{diag}(1, 1, 1, -1, -1)$. Note, however, that all this fields and their VEVs more naturally look in the context of GUTs higher than $SU(5)$. E.g. in $SU(5) \otimes SU(5)$ they can emerge from the mixed representations like $(5, \bar{5})$. Let us now consider the following superpotential terms:

$$\left(\bar{F}_1 F + T_1 \bar{T}_1\right) \Phi + \left(\bar{F}_2 F + T_2 \bar{T}_1\right) \Omega + T_3 \bar{T}_2 \Sigma \quad (23)$$

Then, the VEVs of Φ , Ω and Σ pick up the $L \subset \bar{F}_1$, $d^c \subset \bar{F}_2$, $e^c \subset T_1$, $u^c \subset T_2$ and $Q \subset T_3$ as an ordinary massless quarks and leptons of the SM, while all other states get masses from the couplings in (23). At this stage, keeping only massless states as an external ones and while considering all massive states as those of possible intermediate, one can define five separately conserved global charges. They are:

$$\begin{aligned} C(\bar{F}_1) &= N(e) + N(\nu) \\ C(\bar{F}_2) &= N(d^c) \\ C(T_1) &= N(e^c) \\ C(T_2) &= N(u^c) \\ C(T_3) &= N(u) + N(d) \end{aligned} \quad (24)$$

where N denotes the particle number operator. Now, whatever mechanism is responsible for the generation of the masses for the SM quarks and leptons above, it is evident that \bar{F}_1 and \bar{F}_2 will couple to T_1 and T_3 , respectively and T_2 will couple to T_3 as well. Thus, the only two combinations of charges in (24) will survive as an unbroken ones. Namely,

$$\begin{aligned} Q_1 &= N(e) + N(\nu) + N(e^c) \equiv L \\ Q_2 &= N(u) + N(d) + N(u^c) + N(d^c) \equiv B \end{aligned} \quad (25)$$

separately conserved. They are the lepton L number and the baryon number B . Thus, the proton is stable in all orders of perturbation theory.

The following remark is in order. One could ask, what can be the origin of the "leptonic" fragments $F + \bar{F}$ in big representations of $SU(2) \times U(1)$ considered in previous section, which are needed to properly correct the RG evolution of the gauge couplings. Clearly, these can naturally emerge from the GUT superfields in big representations. For example, consider the $SU(5)$ superfields in 2-index symmetric representations $T_{ab} \sim 15$ and $\bar{T}^{ab} \sim \bar{15}$. One can consider the couplings $T\Phi\bar{T}$ with the Higgs Φ having the VEV $\propto \text{diag}(1, 1, 1, 0, 0)$. Clearly, this would give order M_U mass to all fragments in T and \bar{T} apart of the fragments $F(3, 2) + \bar{F}(3, -2)$ (all indices from $SU(2)$ subgroup). Any other

"leptonic" superfield in representations $F(D, Y) + \overline{F}(D, -Y)$ can be left light by the interaction with the Higgs Φ by proper choice of the corresponding big $SU(5)$ representation, while any fragment with at least one colour index would get mass order M_U . Thus, the extra particle states needed for correcting the RG evolution of the gauge couplings for achieving the TeV scale unification can be obtained by the same "missing VEV" mechanism which also guarantees the fermion mass arrangement rendering the proton stable and also leaves the Higgs doublets light.

4 Conclusions

We have demonstrated that the the unification of $SU(3) \times SU(2) \times U(1)$ gauge couplings at very low scale can be achieved in four dimensions with the help of not so little friends - some extra particles in rather big representations of $SU(2) \times U(1)$ at some intermediate scales between few hundred GeV and TeV. In this case no power like RG evolution has to be invoked in extra dimensions. These particles can be directly observed at the future accelerators and have many phenomenological implications. Theoretical models can be constructed in which the garnd unification occurs at TeV scales but proton remains stable.

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